When $a$ is constant:

\[ x = x_0 + v_0 t + \frac{1}{2} a t^2 \]
\[ v = v_0 + at \]
\[ v^2 = v_0^2 + 2a\Delta x \]
\[ \Delta x = \frac{1}{2}(v_0 + v)t \]

When it isn't:

\[
\frac{d v}{d t} = a = \frac{d^2 x}{d t^2}
\]

Always true:

\[
\int a \, dt = v
\]
\[
\int v \, dt = x
\]
\[ a = \frac{t^2 - 5t^3}{t} \]

What is position at \( t = 1 \) s?

If \( v \circ 2s = 10 \text{m/s} \)

Position at \( 2s = 10 \text{m} \)

\[ v(t) = \int a(t) \, dt \]

\[ = \int \left( \frac{t^2 - 5t^3}{t} \right) \, dt \]

\[ = \frac{1}{5}t^3 - \frac{5}{4}t^4 + v_0 e \]

\[ v(2) = 1 = \frac{1}{3}(2)^3 - \frac{5}{4}(2)^4 + v_0 \]
\[ x = \int v \, dt \]
\[ = \left( \frac{1}{2} t^3 - \frac{5}{4} t^4 + \omega_0 \right) dt \]
\[ \left( x_0 + \frac{1}{2} t^4 - \frac{1}{4} t^5 + \omega_0 t \right) \]
\[ x(t) = 10m \]

\[ \vec{r} = 10t \hat{i} - (50t^2 + \frac{t^3}{2}) \hat{j} \]
\[ \vec{v} = \frac{d\vec{r}}{dt} = 10 \hat{i} - (100t + \frac{3}{2}t^2) \hat{j} \]
\[ \vec{a} = \frac{d\vec{v}}{dt} = 0 \hat{i} - (100 + 2t) \hat{j} \]
Center acceleration / circular motion

\[ a = \frac{d\vec{v}}{dt} \]

\[ \vec{a} = \frac{d\vec{v}}{dt} \]

\[ (\vec{v}_1) = |\vec{v}_1| = v \]

\[ \vec{v}_1 + \Delta \vec{v} = \vec{v} \]

due to speed

\[ \vec{a}_\text{cent} \]

due to centripetal force

\[ \vec{a} \]

due to direction
\[ x = x_0 + v_0 t + \frac{1}{2} at^2 \]

\[ v = v_0 + at \]

\[ \vec{a} = \frac{dv}{dt} \]
\[ y_1 = y_0 - \frac{1}{2} gt^2 \]

\[ y_2 = y_0 - t g (t-1)^2 \]
pj 59, 2.10 ν

\[ \frac{dv}{dt} = 0. \]

\[ a = \frac{\Delta v}{\Delta t} \]

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a

\[ \Delta t \leq \Delta t \]
\[ \vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \]

\[ \vec{A} \cdot \vec{B} = AB \sin \theta \]

R. H. R. \( \vec{A} \times \vec{B} \) direction

\[ \vec{A} \cdot \vec{B} = A (B \cos \theta) \]

\[ = A \parallel B \]
2.96 \ \ \ \ \Delta t = 1s

\[ y = v_0 t - \frac{1}{2} g t^2 \]

\[ y_{\text{max}} = v_0 \left( \frac{1}{2} \right) - \frac{1}{2} g \left( \frac{1}{2} \right)^2 \]

\[ = \frac{1}{2} v_0 - \frac{1}{8} g \]

\[ 2y_{\text{max}} + \frac{1}{4} g = v_0 \]

\[ y = (2y_{\text{max}} + \frac{1}{4} g) t - \frac{1}{2} g t^2 \]

\[ \frac{1}{2} y_{\text{max}} = (2y_{\text{max}} + \frac{1}{4} g) t - \frac{1}{2} g t^2 \]

\[ t^2 (\frac{1}{2} g) + t (2y_{\text{max}} - \frac{1}{4} g) + (\frac{1}{8} y_{\text{max}}) = 0 \]
Had I gone one extra step, I would have realized this is really messy; also, I think now that the "1 sec" is really a ballpark they say as an average, not the actual time in this case. I think we're not supposed to know time. So I'd have gone back a step and said "okay, let's find a simpler approach.

Consider starting at the maximum height: the equation of motion will be y = y_max - 1/2gt^2. To go from the max height to y_max/2 will take \( t_b = \sqrt{\frac{y_{\text{max}}}{g}} \). On the graph above, \( t_b \) will be \( t_2 \) minus the time of the maximum height. The total time the athlete spends above y_max/2 is twice this time, so 2\( \sqrt{y_{\text{max}}/g} \). The time it takes for the athlete to hit the ground is \( t_{\text{grnd}} = \sqrt{2y_{\text{max}}/g} \). The time the athlete spends below y_max/2 is \( t_{\text{grnd}} - t_b \), which is also the amount of time it takes for him to reach it on the other side of the parabola. So the answer to the question is the ratio of these times: \( R = \frac{2t_b}{t_{\text{grnd}} - t_b} = \frac{2}{(t_g/t_b - 1)} = \frac{2}{[\sqrt{2} - 1]} = 4.83 \).
2.98\[\Delta t=1.30s\]
\[a = -9.8\]
\[y_0 = 0\]
\[v_0 = 0\]
\[y_f = -H\]
\[v_f = ?\]
\[y = -\frac{1}{2}gt^2\]
\[y_1 = \frac{2}{3}H = \frac{1}{2}gt_1^2\]
\[y_2 = -H = \frac{1}{2}gt_2^2\]
\[t_2 - t_1 = 1.3\]
\[H = \frac{1}{2}g(t_1 + 1.3)^2\]
\[\frac{2}{3}H = \frac{1}{2}g t_1^2\]
a) I found $t_1 = 1.3/(\sqrt{3}/2 - 1) = 5.78s$
   Sub that back into either equation to find the height $H = 246 m$

b) The above used the positive root when taking the square-root of both sides of $[(t_1+1.3)/t_1]^2 = 3/2$. Taking the negative root leads to $H=2.5m$; this corresponds to the time it would take if it was thrown up from 2.5 m, reach a maximum height, and then fall to the ground in 1.3 s.
A block is launched at 30° above horizontal.

What is v_y0?

\[ v_{x0} = 20 \cos 30° \quad v_{y0} = 20 \sin 30° \]

\[ y(t) = 0 = 20 \sin 30° \cdot t - \frac{1}{2} g t^2 \]

\[ t = \frac{40 \sin 30°}{9.8} \]

\[ x = v_0 \cos 30° \cdot t \]
2.94

\[ y_0 = u_0 \]
\[ y_{1,0} = 0 \]
\[ y_{2,0} = 0 \]
\[ y_{2,0} = h \]

\[ y_1(t) = u_0 t - \frac{1}{2} gt^2 \]
\[ y_2(t) = h - \frac{1}{2} gt^2 \]

\[ y_1(T) = y_2(T) \]
\[ u_0 T - \frac{1}{2} g T^2 = h - \frac{1}{2} g T^2 \]

\[ T = \frac{H}{u_0} \]
\[ \begin{align*}
H &= v_0^2 / g \\
\text{at } t &= v_0 / g \\
\text{when } \frac{d}{dt} v &= -g
\end{align*} \]
Exam I review - prob 4

\[ y_{bal} = H - v_{bal} t - \frac{1}{2} g t^2 \]

\[ h = \frac{H}{t^2} - v_{bal} t = \frac{1}{2} g t^2 \]

\[ t^2 - 2 t v_{bal} + \left( v_{bal}^2 + 2 g (H-h) \right) = 0 \]

\[ a = \frac{1}{2} g \quad b = v_{bal} \quad c = -(H-h) \]

\[ t = \frac{-v_{bal} \pm \sqrt{v_{bal}^2 - 2g (H-h)}}{2g} \]

\[ D = v_{piv}(\ldots) \]
prob 3/ a) \[ a = \frac{dv}{dt} \]

\[ a_x = \frac{dv_x}{dt} = 0.052\ t^2 \]
\[ a_y = \frac{dv_y}{dt} = -9.27 \]

b) \[ x = \int v_x\ dt = 7.3t + \frac{0.026}{3} t^3 \]
\[ y = y_0 - \frac{9.27}{3} t^2 \]

\[ x(t=0) = 5000\ m = x_0 \]
\[ y(t=0) = -450\ m = y_0 \]
\[ y(t) = 5000 \left( 1 - e^{-0.27t} \right) = 0 \]

\[ t = \sqrt{\frac{10000}{0.27}} \]

\[ v_y(t) = \]
Exam I Review

\[ \begin{align*}
\text{Eqns:} & \quad \begin{cases}
y_0 = H \\
y_f = 0 \\
v_0 \sin \theta \\
v_y = ? \\
a_y = -g \quad x_0 = 0 \quad x_f = ? \quad v_{0y} = v_0 \sin \theta \\
v_{bx} = v_0 \cos \theta \\
v_{bc} = v_0 \cos \theta \\
a_x = 0
\end{cases} \\
\end{align*} \]

\[ y(t) = H_0 + v_0 \sin \theta \cdot t - \frac{1}{2} g t^2 = 0 \]

\[ x(t) = v_0 \cos \theta \cdot t \]

\[ x_b(t) = D - v_b t \quad D - v_b t = v_0 \cos \theta \cdot t \]
3.52/ \[ y_0 = 30 \text{m} \]
\[ v_{0y} = 10 \text{m/s} \]
\[ a_y = -g \]
\[ v_{0x} = 15 \text{m/s} \]
\[ a_x = 0 \]
\[ x = 15 \text{(m)} = D \]

\[ y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \]
\[ t = \text{?} \]