A) [4 points] Consider a box of mass $M = 1$ kg being held stationary by three ropes as shown in the figure. The upper ropes make angles of 30 and 45 degrees with the horizontal ceiling as shown. Find the tension in the rope making the 30 degree angle with the ceiling.

\[
\begin{align*}
T_A \cos 45^\circ &= T_B \cos 30^\circ \\
T_A \sin 45^\circ + T_B \sin 30^\circ &= Mg \\
T_A &= 8.78 \text{ N} \\
T_B &= 7.17 \text{ N}
\end{align*}
\]

Short Problems (Circle the correct option) [NO Partial Credit] [20 Points]

B) [4 points] Below is a graph of potential energy, $U$, vs. position, $x$. The total mechanical energy is indicated by the horizontal line $E = K + U$, where $K$ is the kinetic energy. What is the value of the kinetic energy at point $x = B$?

\[
\begin{align*}
E &= K + U \\
K &= \frac{E}{2}
\end{align*}
\]
C) [4 points] The following is a graph of the force on an object as a function of position. Find the work done by this force during a displacement from \( x = -4.00 \) to \( 4.00 \) m

\[
W_F = \int_{-4}^{0} F \, dx + \int_{0}^{4} F \, dx
\]

\[
= 4 \int - 8 \int
\]

\[
= -4 \text{ J}
\]

A) -12.0 J
B) -4.00 J
C) -1.00 J
D) 12.0 J
E) 3.00 J

D) [4 points] Three masses of mass, M, M and 2M are spaced an equal distance D apart along the x axis. Relative to the mass M in the center, find the location of the center-of-mass of this system.

\[
X_{cm} = \frac{M(-D) + M(0) + 2M(0)}{4M}
\]

\[
= \frac{MD}{4M} = \frac{D}{4}
\]

A) D
B) 2D
C) 5/4D
D) 1/4D
E) 3/4D
Problem 2 (20 points)

Two boxes are connected by a weightless cord running over a light frictionless pulley. Box A has mass $M_A = 10.0 \text{ kg}$ and is initially at rest on an incline that makes an angle $\theta = 35$ degrees with respect to the horizontal. The coefficient of kinetic friction between the box and the incline is 0.10. The mass of box B is $M_B = 15.0 \text{ kg}$. The system begins to move just after it is released.

\[ \text{(6 pts) A)} \text{ Draw a free-body diagram for each box, identifying the forces acting on each one.} \]

\[ \text{(See above)} \]

\[ \text{(10 pts) B)} \text{ Determine the acceleration of each box.} \]

\[ \text{For mass A} \quad \Sigma F_{\text{net}x} = -f + T - M_A g \sin \theta = M_A a_x \]

\[ \text{For B} \quad \Sigma F_{\text{net}y} = T - M_B g = M_B a_y \]

\[ \text{with} \quad a_x = -a_y \]

\[ \text{Subtract $M_B$ equation from $M_A$ equation.} \]

\[ -f + T - M_A g \sin \theta - T + M_B g = (M_A + M_B) a_x \]

\[ \text{with} \quad f = \mu_k M_A g \cos \theta \]

\[ a_x = \frac{M_B g - \frac{\mu_k M_A g \cos \theta}{M_A + M_B}}{M_A + M_B} = \frac{3.31 \text{ m/s}^2}{\text{case}} \]

\[ \text{(4 pts) C)} \text{ What is the tension in the cord?} \]

\[ T - M_B g = M_B a_y \]

\[ T = M_B (-a_x + g) = \sqrt{97.4 \text{ N}} \]
Problem 3 (20 points)

A cart with mass \( M = 20 \text{ kg} \) moves through the inside of a vertical loop with a radius of 1 m. The cart passes through the bottom of the loop (point A) at a speed of 15 m/s. When the cart reaches the top of the loop (point B), its speed is 10 m/s.

![Diagram of a vertical loop with point A and point B labeled]

A) How much work is done by non-conservative forces between point A and point B?

\[
W_{\text{FC}} = \Delta ME = \left( \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \right) + (Mgh_f - Mgh_i)
\]

\[\left| W_{\text{FC}} \right| = -858 \text{ Joules}\]

B) Now suppose that the cart has a braking system that leads to a net force of friction of constant magnitude as the cart moves between points A and B. What is the magnitude of this net force of friction?

\[
W_{\text{FC}} = \int f \cdot \mathbf{v} = (f_{\text{Ave}}) \Delta t
\]

\[
f_{\text{Ave}} = \frac{W_{\text{FC}}}{\Delta t} = \frac{-858}{2} = -429 \text{ N}
\]

\[|f_{\text{Ave}}| = 429 \text{ N}\]
(7pt) c) What is the normal force on the cart at point B?

At top

\[-N - Mg = F_{\text{radial}} = -\frac{MV^2}{R}\]

\[N = \frac{MV^2}{R} - Mg\]

\[= 1804 \text{ N downward}\]
Problem 4 (20 points)

A ball of mass $M$ starts from rest at the top of a frictionless ramp, a distance $L$ above a table. The table is also a height $L$ above the ground. The ball slides down and collides elastically with a second ball of mass $3M$ that rests at the edge of the table. Answer the following in terms of the quantities given:

\[ mgL = \frac{1}{2} M v_f^2 \]

\[ v_f = \sqrt{2gL} \]

(4pt) A) Find the velocity of the ball of mass $M$ just before impact with the ball of mass $3M$.

(8pt) B) Find the velocity of the ball with mass $3M$ right after impact.

\[ M v_f = M v_2 + 3M u \]
\[ \frac{1}{2} M v_f^2 = \frac{1}{2} M v_2^2 + \frac{1}{2} (3M) u^2 \]

Factoring out $M$ gives:
\[ v_f^2 = v_2^2 + 3u^2 \]

Solving for $u$ gives:
\[ v_f^2 = v_2^2 + 6v_2 u + 9u^2 \]
\[ 0 = 6v_2 u + 9u^2 \]
\[ v_2 = -u \]

Then:
\[ v_f = -u + 3u = 2u \]

\[ \frac{u}{2} = \frac{v_f}{2} \]
(4pt) C) How long does it take for the ball with mass 3M to hit the floor from impact?

\[ \text{Time to hit the floor} \quad L = \frac{1}{2} gt^2 \]

So

\[ t_{\text{floor}} = \sqrt{\frac{2L}{g}} \]

(4pt) D) What is the "speed" of the ball with mass 3M just before impacting the floor?

\[ |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{v^2 + (-g t_{\text{floor}})^2} \]

\[ = \sqrt{\left(\frac{v_f}{2}\right)^2 + g^2 \left(\frac{2y}{g}\right)^2} = \sqrt{\frac{2gL}{4} + 2gL} \]

\[ = \sqrt{2gL \left(1 + \frac{1}{4}\right)} \]
**Problem 5** (20 points)

A mass $M$ rests against the end of a compressed spring on an incline that makes an angle of $\theta$ with the horizontal. The spring constant is $k$ and the equilibrium position is $x_0$. The coefficient of friction between the block and the incline is $\mu_k$ and the spring is compressed a distance $L$ from its equilibrium length and held with a string. Initially the block is at rest when the string is cut releasing the spring. The mass starts moving when the string is cut. In terms of the quantities given, answer the following:

A) Draw a free body diagram for the block just as the string has been cut.

\[
\text{See above}
\]

B) Find the work done by friction as the block moves up the incline the distance $L$.

(Note, at this point the spring will no longer be in contact with the block.)

\[
W_{\text{friction}} = \overline{f} \cdot \overline{d} = -fL
\]

\[
= -\mu_k M g \cos \theta L
\]
\( W_{friction} = \Delta K + \Delta U = (\frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2) + (0 - \frac{1}{2} k l^2) \\
+ (M g l \sin \theta - 0) \\
- M k m g \cos \theta = \frac{1}{2} m v_f^2 - \frac{1}{2} k l^2 + M g l \sin \theta \\
\frac{1}{2} m v_f^2 = \frac{1}{2} k l^2 - M k m g \cos \theta - M g l \sin \theta \\
V_f = \sqrt{\frac{2}{M} (\frac{1}{2} k l^2 - M k m g \cos \theta - M g l \sin \theta)}