Work-Energy Theorem

These are the problems that you and a team of other 2-3 students will be asked to solve during the recitation session next week. Your team can do better if you think about the approach and explanation for these problems BEFORE coming to class. You may want to follow the solved example on the last pages of this form.

1. Blocks with friction: A block of mass $M_1$ rests on top of a block of mass $M_2$ that rests on a horizontal surface. A light rope attached to $M_2$ is used to pull on it with a force $F$. The coefficient of sliding friction between $M_2$ and the horizontal surface is $\mu_2$. When $M_2$ is pulled (and therefore accelerates), the frictional force between the blocks is not big enough to keep $M_1$ stuck to it, hence $M_1$ slides on $M_2$. The coefficient of kinetic friction between the two blocks is $\mu_1$.
   a) Draw free body diagrams and find the net force on each block.
   b) Find the velocities of the blocks $M_1$ and $M_2$ at the moment when $M_1$ has moved a distance $d$ relative to $M_2$.
   e) Assign appropriate numerical values to the variables in this problem for illustrating all the situations that are possible for this problem.

2. Speeding ticket? You are driving your car uphill along a straight road. Suddenly you see a car run a red light and enter the intersection just ahead of you. You slam on your brakes and skid in a straight line to a stop, leaving skid marks ____ feet long. A policeman observes the whole incident and, much to your shock, gives you a ticket for exceeding the speed limit of ____ mph. When you get home, you consult your Physics 218 notes and estimate that the coefficient of kinetic friction between your tires and the road was ____ and the coefficient of static friction was ___. You estimate that the hill made an angle of ____ degrees with the horizontal. You look up in your owner’s manual and find that your car weighs ____ lb. Will you fight the traffic ticket in court?

Approach: Under this tab, list the steps taken by your team for finding each solution. You answer here the questions WHAT? and HOW?

Approach 1.

Approach 2.
Explanation: Under this tab, explain why your team has chosen those approaches. You answer here the questions WHY? and WHEN?
Explanation 1.

Explanation 2.
Work and Kinetic Energy: Solved Example

Block on Incline: A block of mass 3.0 kg is moved up an incline that makes an angle of 37° with the horizontal under the action of a constant horizontal force of 40 N. The coefficient of kinetic friction between the block and the incline is 0.1. The block is initially at rest. What is the kinetic energy of the block after it has been displaced 2.0 m along the incline?

Conceptual Analysis:
- The block begins with zero kinetic energy; whatever energy it obtains must be equal to the work done on the block.
- Three forces do work on the block: horizontal force, gravitational force, and frictional force.
- Work of a constant force parallel to the displacement is the net force on the block times the distance it is displaced.

Strategic Analysis:
- Find the net force on the block.
- Multiply the net force (constant) by the distance (parallel to the force) to obtain the net work done on the block, and thereby obtain the change in kinetic energy.

Quantitative Symbolic Analysis:
- Begin by labeling the given quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle of incline</td>
<td>θ</td>
<td></td>
</tr>
<tr>
<td>mass of block</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>displacement of block</td>
<td>d</td>
<td></td>
</tr>
<tr>
<td>coefficient of kinetic friction</td>
<td>μ_k</td>
<td></td>
</tr>
<tr>
<td>horizontal force</td>
<td>F_h</td>
<td></td>
</tr>
</tbody>
</table>

- We’ll also use

<table>
<thead>
<tr>
<th>Force</th>
<th>Normal component</th>
<th>Parallel component</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_h</td>
<td>-F sinθ</td>
<td>F cosθ</td>
</tr>
<tr>
<td>F_g</td>
<td>-F_g cosθ = -mg cosθ</td>
<td>-F_g sinθ = -mg sinθ</td>
</tr>
<tr>
<td>F_N</td>
<td>F_N</td>
<td>0</td>
</tr>
<tr>
<td>F_f</td>
<td>0</td>
<td>-F_f</td>
</tr>
<tr>
<td>F_net</td>
<td>0</td>
<td>F_net</td>
</tr>
</tbody>
</table>

We are looking for \( K \) kinetic energy of the block after it has been displaced.

Place all the quantities on the diagram illustrating the situation. Make your choice of axes: perpendicular (normal) direction \( \hat{n} \) and parallel (tangent) direction \( \hat{t} \).

Since the block remains on the incline, we know that the net force in the direction perpendicular to the incline must be zero. The forces perpendicular to the incline are \( F_N \) and a component of both \( F_g \) and \( F \) acting perpendicular to the plane.

Next, we look at the net force parallel to the plane. There are three forces that act parallel to the plane. Determine which component of each force acts parallel to the plane.
Because the sum of the perpendicular forces is zero, the normal reaction force on the box is
\[ F_N = mg \cos \theta + F \sin \theta \]
This allows us to calculate the force of kinetic friction
\[ F_f = \mu_k F_N = \mu_k (mg \cos \theta + F \sin \theta) \]
Now, we can do the addition in the column of the parallel components:
\[ F_{\text{net}} = [F \cos \theta - mg \sin \theta - \mu(mg \cos \theta + F \sin \theta)] \]
We can multiply the net force equation by the net distance moved to find the net work.
\[ W_{\text{net}} = F_{\text{net}} d = [F \cos \theta - mg \sin \theta - \mu(mg \cos \theta + F \sin \theta)] d \]
We also know that \( \Delta K = K_f - K_i = W_{\text{net}} \), with \( K_i = 0 \) because the block starts from rest.

**Quantitative Numerical Analysis:**

- Inserting the numerical values given by the problem:

\[ K = [40 \cos 37^\circ - 3 \times 9.8 \times \sin 37^\circ - 0.1 (3 \times 9.8 \times \cos 37^\circ + 40 \sin 37^\circ)] 2 = 18.99 \text{ N} \times m = 19 \text{ J} \text{ (2 sign figs)} \]

(This problem can also be solved by finding the work done individually by each of the three parallel force components and then adding them together to obtain the result of 19 J.)